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Mathematical studies
Standard level
Paper 1

Tuesday 3 November 2020 (afternoon)

Candidate session number

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1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.



Maximum marks will be given for correct answers. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Answers must be written within the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. Iron in the asteroid *16 Psyche* is said to be valued at 8973 quadrillion euros (EUR), where one quadrillion = 10^{15} .
- (a) Write down the value of the iron in the form $a \times 10^k$ where $1 \leq a < 10$, $k \in \mathbb{Z}$. [2]

James believes the asteroid is approximately spherical with radius 113 km. He uses this information to estimate its volume.

- (b) Calculate James's estimate of its volume, in km^3 . [2]

The actual volume of the asteroid is found to be $6.074 \times 10^6 \text{ km}^3$.

- (c) Find the percentage error in James's estimate of the volume. [2]

Working:

Answers:

- (a)
- (b)
- (c)



2. Olava’s Pizza Company supplies and delivers large cheese pizzas.

The total cost to the customer, C , in Papua New Guinean Kina (PGK), is modelled by the function

$$C(n) = 34.50n + 8.50, n \geq 2, n \in \mathbb{Z},$$

where n , is the number of large cheese pizzas ordered. This total cost includes a fixed cost for delivery.

- (a) State, in the context of the question,
 - (i) what the value of 34.50 represents;
 - (ii) what the value of 8.50 represents. [2]
- (b) Write down the minimum number of pizzas that can be ordered. [1]

Kaelani has 450 PGK.

- (c) Find the maximum number of large cheese pizzas that Kaelani can order from Olava’s Pizza Company. [3]

Working:

Answers:

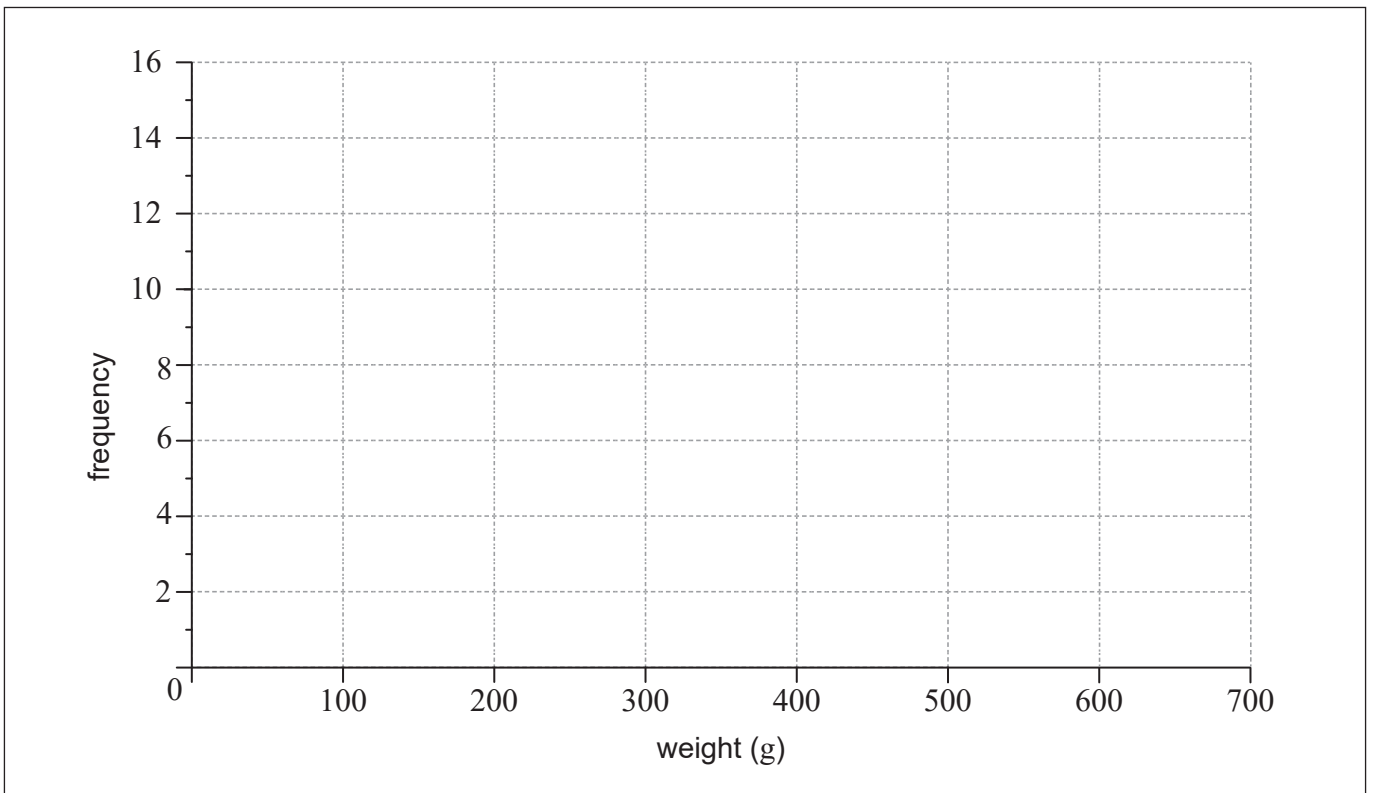
- (a) (i)
- (ii)
- (b)
- (c)



3. Hafizah harvested 49 mangoes from her farm. The weights of the mangoes, w , in grams, are shown in the following grouped frequency table.

Weight (g)	$100 \leq w < 200$	$200 \leq w < 300$	$300 \leq w < 400$	$400 \leq w < 500$	$500 \leq w < 600$
Frequency	4	7	14	16	8

- (a) Write down the modal group for these data. [1]
- (b) Use your graphic display calculator to find an estimate of the standard deviation of the weights of mangoes from this harvest. [2]
- (c) On the grid below, draw a histogram for the data in the table. [3]



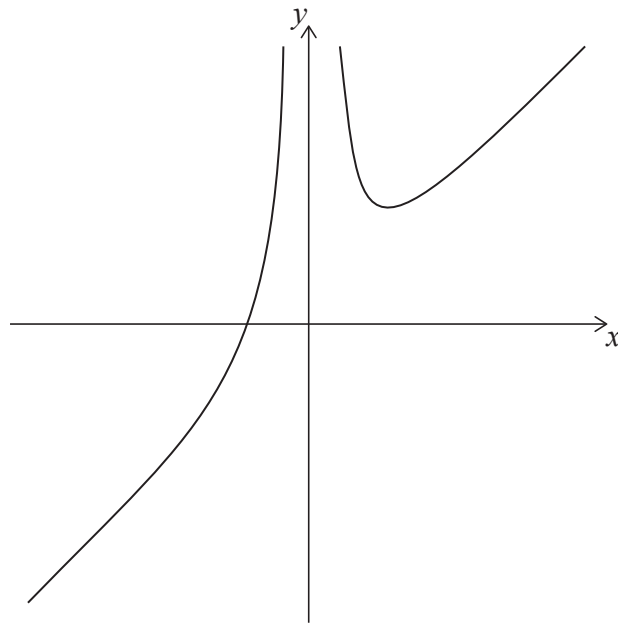
Working:

Answers:

- (a)
- (b)



4. Consider the graph of the function $f(x) = x + \frac{12}{x^2}$, $x \neq 0$.



(a) Write down

(i) the zero of $f(x)$;

(ii) the coordinates of the local minimum point.

[4]

Consider the function $g(x) = 3 - x$.

(b) Solve $f(x) = g(x)$.

[2]

Working:

Answers:

(a) (i)

(ii)

(b)



5. On a trip to Spain, Joe changed 3400 United States dollars (USD) into euros (EUR) at a rate of $1 \text{ USD} = 0.8550 \text{ EUR}$. There was no commission charged.

(a) Calculate the **exact** amount of EUR that Joe received. [2]

During the trip, Joe spent a total of 1473 euros. He then changed the remaining euros back into United States dollars. No commission was charged.

The exchange rate was $1 \text{ USD} = 0.8494 \text{ EUR}$.

(b) Calculate the amount of USD that Joe received. Give your answer correct to two decimal places. [2]

In Spain, Joe paid 1077 euros for a gold coin weighing 31.104 grams.

At the time of purchase, the price of one gram of gold was x EUR.

(c) Find the value of x . [2]

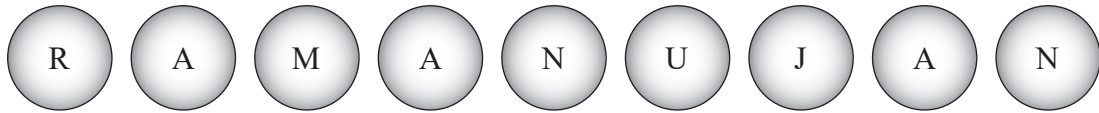
Working:

Answers:

- (a)
- (b)
- (c)



6. Srinivasa places the nine labelled balls shown below into a box.



Srinivasa then chooses two balls at random, one at a time, from the box. The first ball is **not replaced** before he chooses the second.

- (a) Find the probability that the first ball chosen is
 - (i) labelled A;
 - (ii) labelled A or labelled N. [2]
- (b) Find the probability that the second ball chosen is labelled A, given that the first ball chosen was labelled N. [2]
- (c) Find the probability that both balls chosen are labelled N. [2]

Working:

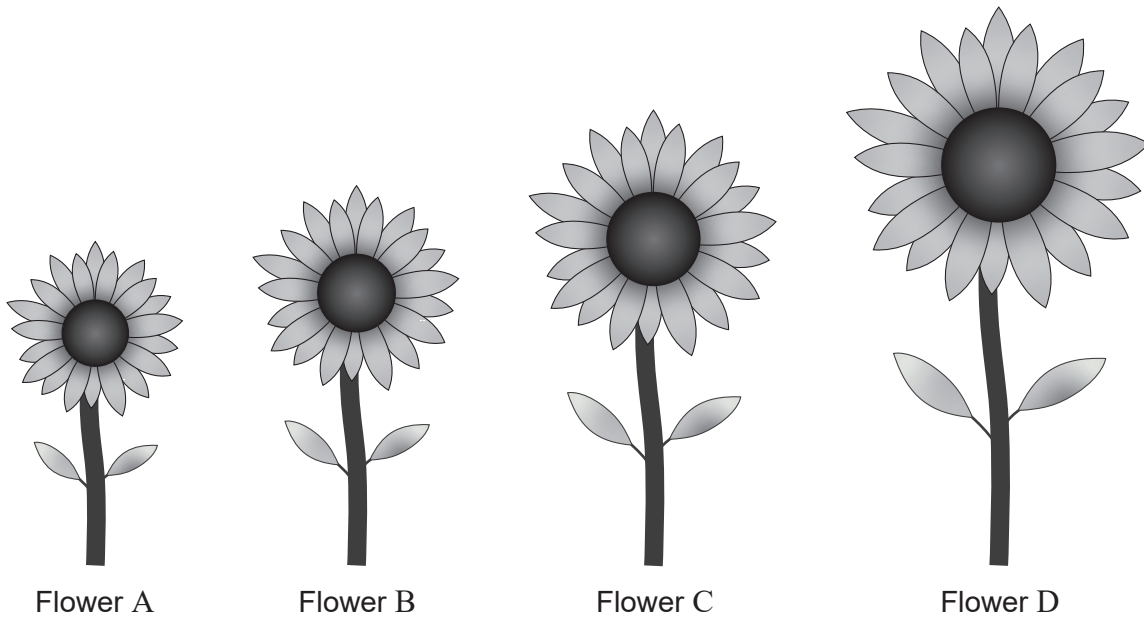
Answers:

- (a) (i)
- (ii)
- (b)
- (c)



7. Anne-Marie planted four sunflowers in order of height, from shortest to tallest.

diagram not to scale



Flower C is 32 cm tall.

The median height of the flowers is 24 cm.

(a) Find the height of Flower B. [2]

The range of the heights is 50 cm. The height of Flower A is p cm and the height of Flower D is q cm.

(b) Using this information, write down an equation in p and q . [1]

The mean height of the flowers is 27 cm.

(c) Write down a second equation in p and q . [1]

(d) Using your answers to **parts (b) and (c)**, find the height of

(i) Flower A;

(ii) Flower D. [2]

(This question continues on the following page)



(Question 7 continued)

Working:

Answers:

(a)

(b)

(c)

.....

(d) (i)

(ii)



8. Give your answers in this question correct to the nearest whole number.

Imon invested 25 000 Singapore dollars (SGD) in a fixed deposit account with a nominal annual interest rate of 3.6%, compounded **monthly**.

(a) Calculate the value of Imon's investment after 5 years. [3]

At the end of the 5 years, Imon withdrew x SGD from the fixed deposit account and reinvested this into a super-savings account with a nominal annual interest rate of 5.7%, compounded **half-yearly**.

The value of the super-savings account increased to 20 000 SGD after 18 months.

(b) Find the value of x . [3]

Working:

Answers:

(a)

(b)



9. Consider the two propositions p and q .

p : I get the job
 q : I have experience

(a) Write down in words $p \leftrightarrow \neg q$. [2]

(b) Complete the following truth table. [2]

p	q	$\neg q$	$p \leftrightarrow q$	$p \leftrightarrow \neg q$	$(p \leftrightarrow q) \wedge (p \leftrightarrow \neg q)$
T	T	F	T		
T	F	T	F		
F	T	F	F		
F	F	T	T		

(c) (i) Determine whether the compound statement $(p \leftrightarrow q) \wedge (p \leftrightarrow \neg q)$ is a tautology, contradiction or neither.

(ii) Justify your answer to **part (c)(i)**, with reference to the truth table. [2]

Working:

Answers:

(a)

.....

.....

.....

(c) (i)

(ii)



10. On 90 journeys to his office, Isaac noted whether or not it rained. He also recorded his journey time to the office, and classified each journey as short, medium or long.

Of the 90 journeys to the office, there were 3 short journeys when it rained, 22 medium journeys when it rained, and 15 long journeys when it rained. There were also 14 short journeys when it did not rain.

Isaac carried out a χ^2 test at the 5% level of significance on these data, looking at the weather and the types of journeys.

- (a) Write down H_0 , the null hypothesis for this test. [1]
- (b) Find the expected number of short trips when it rained. [3]

The p -value for this test is 0.0206.

- (c) State the conclusion to Isaac's test. Justify your reasoning. [2]

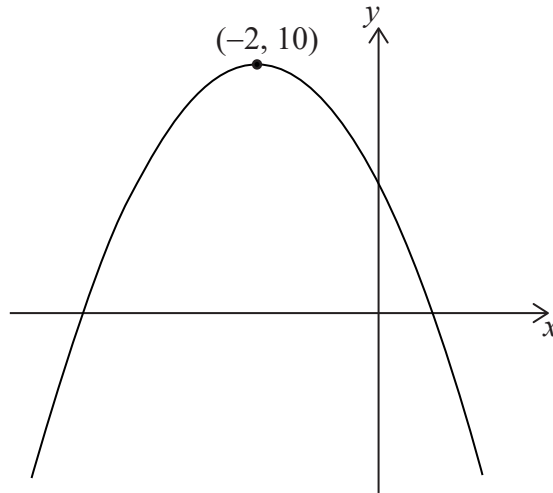
Working:

Answers:

- (a)
- (b)
- (c)
.....



11. The diagram shows the graph of the quadratic function $f(x) = ax^2 + bx + c$, with vertex $(-2, 10)$.



The equation $f(x) = k$ has two solutions. One of these solutions is $x = 2$.

- (a) Write down the other solution of $f(x) = k$. [2]
- (b) Complete the table below placing a tick (\checkmark) to show whether the unknown parameters a and b are positive, zero or negative. The row for c has been completed as an example. [2]

	positive	zero	negative
a			
b			
c	\checkmark		

- (c) State the values of x for which $f(x)$ is decreasing. [2]

Working:

Answers:

(a)

(c)

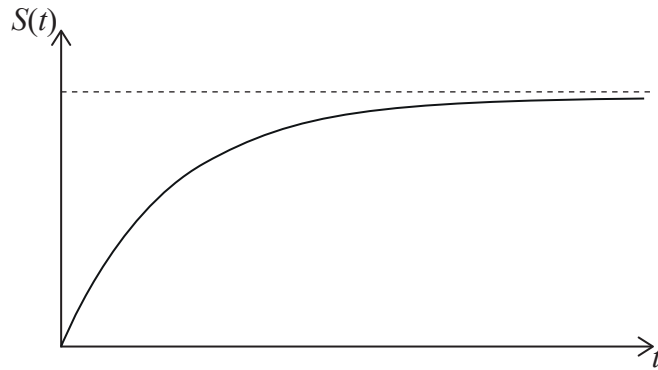


12. Jean-Pierre jumps out of an airplane that is flying at constant altitude. Before opening his parachute, he goes through a period of freefall.

Jean-Pierre's vertical speed during the time of freefall, S , in m s^{-1} , is modelled by the following function.

$$S(t) = K - 60(1.2^{-t}), \quad t \geq 0$$

where t , is the number of seconds after he jumps out of the airplane, and K is a constant. A sketch of Jean-Pierre's vertical speed against time is shown below.



Jean-Pierre's initial vertical speed is 0 m s^{-1} .

- (a) Find the value of K . [2]
- (b) In the context of the model, state what the horizontal asymptote represents. [1]
- (c) Find Jean-Pierre's vertical speed after 10 seconds. Give your answer in km h^{-1} . [3]

Working:

Answers:

- (a)
- (b)
- (c)



13. Consider the graph of the function $f(x) = x^2 - \frac{k}{x}$.

(a) Write down $f'(x)$. [3]

The equation of the tangent to the graph of $y = f(x)$ at $x = -2$ is $2y = 4 - 5x$.

(b) Write down the gradient of this tangent. [1]

(c) Find the value of k . [2]

Working:

Answers:

(a)

(b)

(c)



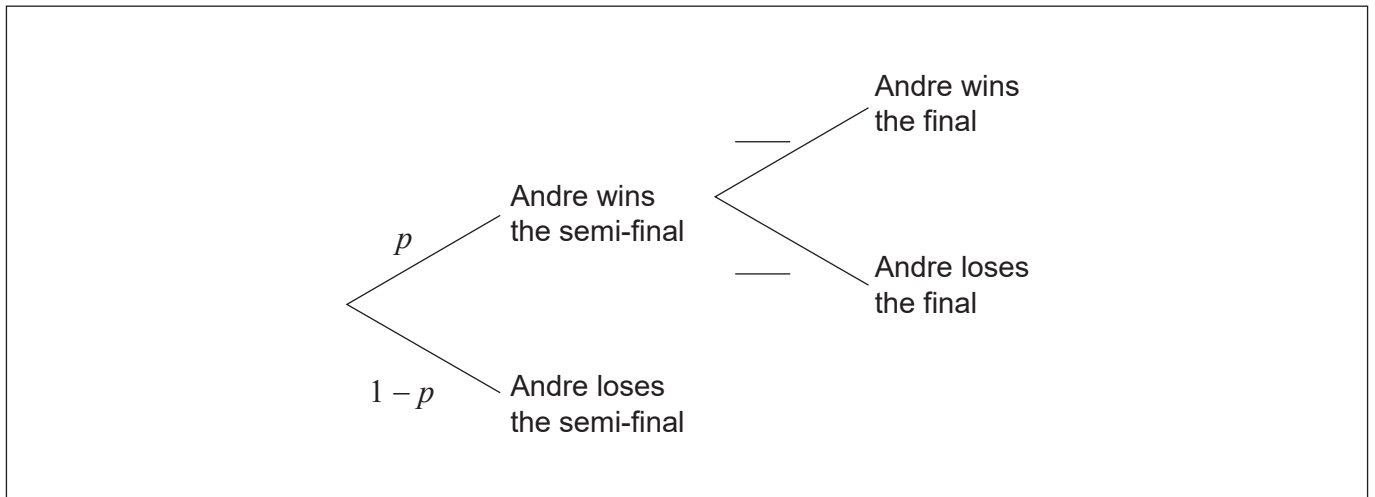
14. Andre will play in the semi-final of a tennis tournament.

If Andre wins the semi-final he will progress to the final. If Andre loses the semi-final, he will **not** progress to the final.

If Andre wins the final, he will be the champion.

The probability that Andre will win the semi-final is p . If Andre wins the semi-final, then the probability he will be the champion is 0.6.

(a) Complete the values in the tree diagram. [1]



The probability that Andre will not be the champion is 0.58.

(b) Find the value of p . [2]

(c) Given that Andre did not become the champion, find the probability that he lost in the semi-final. [3]

Working:

Answers:

(b)

(c)



15. Mia baked a very large apple pie that she cuts into slices to share with her friends. The smallest slice is cut first. The volume of each successive slice of pie forms a geometric sequence.

The second smallest slice has a volume of 30 cm^3 . The fifth smallest slice has a volume of 240 cm^3 .

- (a) Find the common ratio of the sequence. [2]
- (b) Find the volume of the smallest slice of pie. [2]

The apple pie has a volume of $61\,425 \text{ cm}^3$.

- (c) Find the total number of slices Mia can cut from this pie. [2]

Working:

Answers:

- (a)
- (b)
- (c)



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